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LETTERS AND COMMENTS

Electrostatic charges in $v \times B$ fields: with or without special relativity?

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Abstract
In relation to Bringuier's paper (2003 *Eur. J. Phys.* **24** 21), it is pointed out that one cannot always neglect relativistic effects at low velocities. Also, some flaws are identified in his analysis of fields and charges in the case of a circular loop rotating in a constant magnetic field.

A recent analysis of electrostatic charges in $\boldsymbol{v} \times \boldsymbol{B}$ fields presented by Bringuier (2003) contains a starting assumption which perhaps deserves a short comment. According to Bringuier, one need not invoke special relativity when discussing a first-order theory of conducting non-magnetic media that move in magnetic fields. The statement is, of course, perfectly sound as regards the polarization in a moving non-magnetic medium which is given by

$$\boldsymbol{P} = \varepsilon_0(\varepsilon_r - 1)(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}). \tag{1}$$

It seems, however, that without special relativity one would hardly recall that the constitutive equation for the magnetization in the moving medium is

$$\boldsymbol{M} = \boldsymbol{P} \times \boldsymbol{v} \tag{2}$$

in the first-order theory (Rosser 1964, Redžić 2002), and that, consequently, the corresponding Maxwell equation for the curl of $\boldsymbol{B}$ is

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \nabla \times \boldsymbol{M}, \tag{3}$$

where $\boldsymbol{J} = \varrho \boldsymbol{v} + \boldsymbol{J}_c$ is the total, convection plus conduction, current density.

True, for a rigid isolated axisymmetric conductor that rotates uniformly in a net magnetic field (applied plus that induced by convection currents) that is symmetrical about the axis of rotation, the magnetization $\boldsymbol{M}$ vanishes since $\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}$ vanishes. This follows from Ohm's law in differential form, $\boldsymbol{J}_c = \sigma(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$, and from the fact that the conduction current vanishes under steady-state conditions, as Lorrain (1990) and Bringuier (2003) have pointed out. (Note that the vanishing of $\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}$ is obtained in the framework of the corresponding relativistic theory by neglecting the inertia of the conduction electrons (compare Grøn and Vøyenly (1982) and Redžić (2002)).)

However, if the rotating conductor is not isolated, i.e. if flow of current is possible into or out of it, the magnetization in the conductor does not vanish and is given by

$$\boldsymbol{M} = \frac{\varepsilon_0(\varepsilon_\mathrm{r} - 1)}{\sigma} \boldsymbol{J}_\mathrm{c} \times \boldsymbol{v}. \tag{4}$$

In the model discussed by Lorrain (1990) of the Faraday disc connected to a stationary circuit through sliding contacts, the curl of $\boldsymbol{M}$ vanishes. Nevertheless, in the view of the present author, in the general case the magnetization in moving non-magnetic conductors should not be ignored when considering electrostatic charges in $\boldsymbol{v} \times \boldsymbol{B}$ fields.

The neglect of special relativity leads to a wrong solution (wrong if special relativity is correct) to a simple electrodynamical problem even in the zero-velocity limit, in conflict with intuition, as discussed elsewhere (Bartocci and Mamone Capria 1991a, 1991b, Rosser 1993, Redžić 1993). In addition, nowadays it seems to be conclusively demonstrated that special relativity is indispensable in the analysis of magnetic dielectrics that move at low velocities in magnetic fields (cf a recent version using a modern magnetic material (Hertzberg *et al* 2001) of the Wilson–Wilson experiment).

Another point in Bringuier's paper (2003) is perhaps worth noting. The author discussed, *inter alia*, the case of a thin circular non-magnetic conducting loop of wire of uniform circular cross-section, rotating around a diameter in a static uniform magnetic field perpendicular to that diameter. Unfortunately, his analysis of fields and charges in the classic physical system is presumably wrong for the following reasons.

First, Bringuier argues that the bulk equation div $(-\varepsilon_0(\boldsymbol{v} \times \boldsymbol{B})) = \varrho$, where ϱ is the density of free charge, implies what he calls the continuity condition given by his equation (16). However, that argument contains a pitfall: the bulk equation, while mathematically correct, has no physical meaning outside the wire. The correct boundary condition involving the surface free-charge density is that required by div$\boldsymbol{D} = \varrho$. (Compare Sommerfeld's solution (1952) to the problem of a long conducting bar moving along its axis in a uniform static magnetic field perpendicular to the axis.)

Second, there is an omission in Bringuier's expression for the uniform transverse field in an infinite cylinder due to a surface charge distribution over the cylinder; a factor of two is missing. After this omission is corrected, it is clear that his conclusion that '. . . $\boldsymbol{E}$ exactly cancels the radial component. . . of $\boldsymbol{v} \times \boldsymbol{B}$' is wrong.

Third, but most important, in the configuration that was analysed by the author (the ring is rotating around a diameter, in a uniform static magnetic field perpendicular to that diameter), even a uniformly rotating ring does not give rise to a stationary situation. Recall that steady-state situations discussed by Lorrain (1990, 2001) and Redžić (2001, 2002) are possible when an axisymmetric and *rotationally invariant* conductor rotates uniformly in an axisymmetric time-independent magnetic field; the rotation axis and the field and the conductor symmetry axes should coincide, of course. (Obviously, a steady-state situation with a rotating toric wire would be possible only if the wire rotated uniformly around the axis of symmetry perpendicular to the equatorial plane of the wire (Sommerfeld 1952).) Clearly, the thin toric wire rotating around a diameter, discussed by Bringuier, is not a rotationally invariant system. In addition, the wire is the seat and the carrier of time-varying conduction and convection currents. The currents give rise to time-varying induced electric and magnetic fields, and the curl of the electric field is given by the differential form of Faraday's induction law. As can be seen, neglecting the induced electric and magnetic fields, as the author did, we neglect the self-inductance of the wire, a rather rough approximation. Moreover, a more detailed analysis reveals that Bringuier's equation (7) does not apply to the rotating ring discussed in his section 5.

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